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Building a corporate culture in a team

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Abstract

This paper examines how a manager influences preferences and culture in closed organizations where members share a deep, conforming culture. We propose a model for multi-task team agency relationships, where workers balance substitute tasks, each linked to specific preferences. The manager aims to implement a desired corporate culture through costly interventions targeting individuals, framing the issue as a dynamic programming challenge. Our analysis reveals that when the manager seeks a heterogeneous culture, the resulting corporate culture remains sub-optimal, consistently showing less diversity than the desired "first-best" target. This underscores the challenges of cultural shaping and the pressure for uniformity.

Keywords: Culture, Cognitive dissonance and conformity, Dynamic programming, Preference evolution.

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1 Introduction

The building of an organizational culture plays a pivotal role in the long-term success and sustainability of any long-lived group or organization engaged in continuous activities. The characteristics of this culture are closely linked to the welfare and satisfaction of its members, as well as to the organization's overall performance (see Ellison and Mullin, 2014; Kotter and Heskett, 1992; Nadler and Tushman, 1997; Schein, 1985; Schwartz and Davis, 1981; see also, at society level, Fernandez, 2008, 2013; Guiso et al., 2006; Landes, 1998). In recent economic literature, culture is often defined as the distribution of beliefs, preferences, or personal norms within a group or society (see, for instance, Alesina and Giuliano, 2015; Van den Steen, 2010a; Van den Steen, 2010b). From an economic perspective, the need for an appropriate organizational culture enhancing performance originates from the inherent limitations of explicit incentives, such as formal contracts, which are often incomplete or unfeasible. In such contexts, individual preferences and their distribution within the organization become critical determinants of behavior and outcomes. This is why management invests significant resources in building and shaping corporate culture, as it serves as a mechanism to align individual and organizational goals in the absence of perfect contractual solutions.

Our study examines how managers build a corporate culture while considering two key challenges. First, the impossibility of imposing behavior solely through monetary incentives. This understanding opens the door to exploring alternative, socially-driven strategies –such as mentoring and coaching– to shape the team's preferences, rather than merely offering financial rewards[1]. Second, the manager also acknowledges the presence of a deep underlying organizational culture, understood as a set of psychological variables shared among group members, that influences the evolution and long-term distribution of preferences and behaviors. The manager's objective is to shape a corporate culture characterized by a desired set of shared preferences across the group.

We propose a model that applies to multi-task team agency relationships, where workers balance substitute tasks, each having their own preference for a particular mix of these tasks[2] In this paper, a multi-task team is a team where its members have to allocate their time or effort between two substitute tasks. Workers select

[1]As highlighted by agency theory, monetary incentives alone are often insufficient for aligning incentives (see Prendergast, 2007).

[2]Some workers are better prepared or better disposed to one particular mix of these tasks. For example, a professor has triple responsibilities: teaching, research, and administration.

a task combination based on their individual preferences, but their choices are also influenced by peer behavior due to *conformity* –the tendency to align one's actions or preferences with those of others in the group. Over time, worker preferences evolve due to two key factors: (1) *time consistency* –the tendency to align preferences with past and present actions, and (2) the influence of the manager's guidance and activities. The manager acts as a shaper of preferences, working to influence the group by investing resources to instill a desired corporate culture. Our focus is not on how the manager changes an individual worker's preference, but rather on how he shapes the overall distribution of preferences within the group. The manager has a forward-looking plan: to implement a specific set of preferences across the group. We take this ideal culture as given and focus on analyzing the manager's ability and limits in achieving his plan. The manager selects costly interventions aimed at individual members, making this a dynamic programming problem. Therefore, his goal is to minimize the average distance between each worker's preferences and the target preferences while managing the costs of these interventions, which depend on the "cultural distance" between the worker's current and desired preferences.

An example that illustrates the situation we are analysing is professional football. All players in modern football have to display a good disposition to defend and to attack. But obviously, not every member of the team does so in the same proportion. Each player has an individual preference, an inherent predisposition to focus more on attacking or defending. The player's action is the proportion of time they devote to each task during a match. Both actions and preferences are best represented as choices from a continuous set, such as the unit interval $[0, 1]$, where 0 represents "only defend" and 1 "only attack." The coach has an optimal team strategy in mind and works to instill in each player a specific balance of these tasks. Through training, mentoring, coaching, and other costly efforts, the coach gradually adjusts each player's preferences toward the desired mix of attacking and defending. Professional football also illustrates that in multi-task teams, the manager's optimal plan is often to achieve heterogeneity, that is, to create diversity in the individual balances between the two tasks.

We assume in this paper that all members of the organization share the same levels of conformism and consistency. We are going to denote this situation as the deep culture of the members of the organization. It is helpful to think of culture as having two levels, which differ in terms of their resistance to change. At a deeper and less visible level, culture refers to the values shared by the people in a group that tend to persist over time. In our model, workers share the same "deep culture" concerning their mix of conformism and consistency, while they might differ in their preferences concerning the mix between the two tasks in our multi-task agency situation. This latter level of culture evolves at a very different speed than the deep one.

Our main result is that, provided the workers' deep culture displays positive levels of conformity and consistency, and if the manager desires a heterogeneous culture, the steady-state corporate culture will always be suboptimal, a "second best". In particular, the organization will always be less diverse or heterogeneous than the "first best" corporate culture intended by the manager.

We start by showing, as a by-product of our model, that in the absence of the manager's influence, a closed group where all members display positive levels of conformity and consistency tends to reach homogeneity in both preferences and behavior. In addition, the organization's steady state is determined solely by the initial distribution of preferences.

The general case addresses the dynamic programming problem faced by the manager of how to optimally influence workers' preferences over time. We provide a formal characterization of the manager's optimal policy, which helps us to understand the steady-state outcome for each worker. This outcome is a convex combination of two factors: the manager's specific target preference for that worker and the average target preference for the entire organization. We examine how the manager's desired level of organizational heterogeneity, his intertemporal discount factor, and the cost function parameters affect the long-run corporate culture. These factors also influence the per capita effort and total costs needed to sustain the steady-state culture. The gap between the manager's ideal for each worker and the actual steady-state outcome increases with higher levels of conformity and consistency. As a result, the manager achieves less overall success, and the resulting culture is less diverse than intended, with even less diversity in the actions of workers. Interestingly, manager's per capita effort decreases in the steady-state, but the aggregate cost rises due to the greater cultural distance between workers and the manager's target.

A key insight is that both conformism and consistency are crucial for shaping the resulting steady-state corporate culture. Neither factor on its own can fully challenge or hinder the manager's efforts. If there is only conformity or only consistency within the organization, the manager will be able to fully implement his desired preferences for each worker. However, when both conformity and consistency are present, and if the manager aims to create a heterogeneous culture (where workers have varying preferences), we encounter an "impossibility result": the steady-state corporate culture is different from the manager's target, it is less diverse or heterogeneous.

Our paper is related to several strands of the literature.

Firstly, our model is related to the few existing formal studies analyzing corporate culture in a dynamic setting (see, Rob and Zemsky, 2002, Kandori, 2003, and Calabuig, Olcina and Panebianco, 2018). These are "effort" models and their main goal is to analyze the interplay between norms and monetary incentives. However, none of them analyzes the interplay between management and corporate culture, nor do they include the role of a manager as a shaper of preferences, or as a builder of organizational cultures. Our model, to the best of our knowledge, is the first model that analyzes a team multi-task agency relationship in the absence of material incentives and where the principal, i.e., the manager, purposely influences and shapes the team distribution preferences.

More closely related to our dynamic approach is the literature on cultural transmission and, in particular, on the role of cultural leaders. An initial study in this strand of literature is offered by Cordes, Richerson, McElreath and Strimling, 2008. These authors analyze how an entrepreneur can shape human behavior within a firm. They model it as biased transmission of cultural contents via a social learning process. These authors take the entrepreneur's charisma as a given parameter. In other words, they do not analyze the coaching and socialization activities of the entrepreneur as we do.

There is also a recent study on cultural transmission of preferences where the influence of "cultural leaders" is added to parental transmission and oblique transmission which are the usual channels for intergenerational preference evolution (Verdier and Zenou, 2015, 2018; see also Prummer and Sieldlwrek, 2017). Cultural leaders in these models compete for oblique socialization in the inter-generational transmission of a particular preference trait. However, there are some important differences between these models and our model. For instance, these studies work with two preference traits while we work with a continuum of traits or preferences. Another major difference is that the cultural leader tries to spread a particular trait in the population. In our model a manager pursuing a homogeneous organization in preferences is just one particular case. We study the more general case in which the manager might desire some diversity in the long-run organization preference distribution. Finally, these studies look at intergenerational cultural transmission (vertical and oblique). Our model addresses intragenerational cultural change where the psychological driving forces are conformity and consistency (see also, in a different setting, Gravilets, 2021 Calabuig et al., 2018). A closely related paper to our paper is Olcina, Panebianco and Zenou, 2024, where the driving forces of the dynamics are conformity and consistency but there is no manager or leader and, the focus is placed on the network structure of the group.

The rest of the paper is organized as follows. In Section 2, we present the following subsections: 2.1 the model with the stage game, 2.2 the evolution of preferences, and 2.3 the dynamic optimization problem faced by the manager. Section 3 covers the following subsections: 3.1 the Nash equilibrium of the workers; 3.2 the study of the steady state under the absence of the manager; 3.3 the optimal dynamic policy of the manager; and 3.4 the steady-state corporate culture. Sections 4 and 5 measure the success of the manager and how much he should invest to maintaining the steady-state corporate culture. Section 6 provides the impact of the deep culture on the resulting long-term corporate culture and the efforts and costs required to maintain it. We close with the concluding remarks in Section 7.

2 The model

Let us consider an organization composed of a manager and a finite group of workers of fixed size N. Each worker $i \in \{1, \dots, N\}$, at period $t = 0, 1, 2, \dots$ holds a preference[3] $S_i^t \in [0, 1]$, and chooses simultaneously an action $x_i^t \in [0, 1]$. We introduce $x^t = (x_1^t, \dots, x_N^t)$ and $S^t = (S_1^t, \dots, S_N^t)$ as the respective vectors of actions and preferences at period t. The manager holds an ideal preference distribution for the organization's members, represented by a vector of preferences $\bar{S} = (\bar{S}_1, \dots, \bar{S}_N)$, with each element targeted at a specific worker. This preference distribution is assumed to be time-independent, offering an optimal framework for the firm. While the manager doesn't directly dictate the actions x_i^t taken by workers, he influences the evolution of workers' preferences S_i^t to align with the manager's target distribution across the organization.

The timing of the interaction between workers and the manager is the following. Initially, workers engage in an N-player game with complete information, where their utility function integrates their innate preferences and the prevailing group culture. Preferences and behaviors are initially set based on individual innate preferences and the existing group culture. Over time, workers' preferences gradually evolve as individuals interact and are influenced by their own experiences and by the manager. The manager implements a costly strategy aimed at guiding the group towards organizational objectives. Consequently, behaviors may also change as a result of evolving preferences and the indirect effect on the corporate culture. The evolving nature of preferences and behaviors, coupled with the manager's efforts, transforms

[3]We can interpret S_i^t as the ideal action of worker i at period t. Another interpretation, in a different setting, is that S_i^t is an opinion and x_i^t is a declared opinion or stance.

the situation into a dynamic programming problem. Next, we outline the stage game played by the workers and examine how their preferences evolve. Finally, we illustrate the dynamic optimization problem that the manager addresses.

2.1 The stage game of the workers

When a worker takes an action x_i^t at each period t, her behavior is driven by two competing motives: she wants her behavior to agree with her personal preference S_i^t and she also wants it to be as close as possible to the average behavior of the corporation. Hence, the instantaneous payoff function of each worker is a loss function with two components[4],

$$u_i^t(x^t) = -\omega(x_i^t - \langle x^t \rangle)^2 - (x_i^t - S_i^t)^2, \qquad i = 1, \ldots, N. \tag{1}$$

where $\langle x^t \rangle$ is the average action[5] at period t. The first term implies that individuals are conformists and incur costs associated with choosing actions that diverge from social norms modeled as the average action of the peer. The parameter $\omega \geq 0$ describes the taste for conformity[6] and measures the intensity of the social (endogenous) norm for the agents. The second term establishes that individuals derive utility from choosing actions close to their intrinsic "bliss point" S_i^t. It captures the inner discomfort of behaving differently to your preference.

We assume that N is sufficiently large so that each agent has a negligible impact on the group's average action, $\langle x^t \rangle$, and thus perceives it as exogenously given. In other words, while the norm, the average action of the group, is endogenously

[4]In the literature we have two seminal papers related to the shape of this individual utility function. First, in the seminal paper of Akerlof (1997), there is a balance between a loss of utility when the action falls behind everyone else. In addition, there is also a loss that depends on the distance from their intrinsic value. In their paper, Morris and Shin (2002) outline a utility function that consists of two components within the loss function for each individual i. The first component is a standard quadratic loss, which evaluates the distance between the individual's action and the underlying state. The second component, referred to as the "beauty contest," introduces an externality where individuals strive to anticipate the decisions of others within the economy.

[5] Hereafter, for each function f taking value f_i for worker i we define the average value $\langle f \rangle$ as:

$$\langle f \rangle \equiv \frac{1}{N} \sum_{j=1}^{N} f_j.$$

[6]This is the standard way to model conformity and social norms in economics (see Bernheim, 1994; Kandel and Lazear, 1992; Akerlof, 1997; Fischer and Huddart, 2008; Huck, Kubler and Weibull, 2012).

determined, it remains unaffected by the behavior of any single, even powerful, agent.

2.2 Evolution of preferences of the workers

Workers' preferences S_i^t evolve over time due to two psychological sources of change. On the one hand, each worker's preference shifts towards their actual behavior, creating a self-persuasion or cognitive dissonance force[7], which we refer to as "consistency". On the other hand, each worker's preference also adjusts towards the manager's target for that worker, creating a force[8] we term "follow the manager".

According to consistency when there is a discrepancy between the preference and the current behavior of a worker i, her preference S_i^t gradually evolves over time in the direction of her actual behavior following this equation:

$$\hat{S}_i^t = \gamma x_i^t + (1-\gamma) S_i^t, \qquad i = 1, \ldots, N. \tag{2}$$

We denote as $\hat{S}^t = (\hat{S}_1^t, \ldots, \hat{S}_N^t)$ the vector of interim preferences at period t prior to any influence from the manager. These interim preferences are a convex combination of the action and the preference in period t. The parameter $\gamma \in [0, 1)$ measures the level of consistency of all workers. We assume in this paper that the levels of conformity and consistency are shared by all workers and is fixed. This is what we denote as the worker's deep culture. One might attribute this to cultural tightness (Gelfand et al, 2011), which is a cultural attribute exogenous to the organization. Therefore, we might think of our workers coming from a society with a tight culture: many strong norms and high intolerance to deviant behavior and consequently, individuals being quite homogeneous in their "deep" psychological traits.

The other dynamic force that influences workers' preference evolution is the tendency to "follow the leader or follow the authority". We write the evolution over

[7] This source for preference change has a robust psychological foundation and has been widely used not only in this discipline but also in economics (see for instance, Akerloff, 1982 and more recently, Kuran and Sandholm, 2008, Nordblom and Zamac, 2012, Calabuig, Olcina and Panebianco, 2018).

[8] Evolutionary biologists such as Tooby and Cosmides (1992) and Krause and Ruxton (2002) have emphasized that leadership is likely to have evolved in humans to solve coordination problems (see also, Van Vugt et al. 2006). According to Van Vugt et al. (2006) "Leadership originally emerged to solve simple coordination problems in group-living species and has an ancient phylogenetic history. Among humans, leadership was co-opted to deal with specific problems associated with living in large groups. Group decision-making would be facilitated by the emergence of some form of leadership, whereby some individuals persuaded others to follow them in the direction of a preferred waterhole or hunting ground".

the period t of the preferences S_i^t of the members of the organization as the convex combination of the manager objective and the interim preference,

$$S_i^{t+1} = e_i^t \bar{S}_i + (1 - e_i^t)\hat{S}_i^t, \qquad i = 1, \ldots, N. \tag{3}$$

depending on the effort made by the manager $e_i^t \in [0, 1]$.

This way of modelling the influence of the manager as a "cultural parent" is similar to others in the preference transmission literature (see, for example, Bisin and Verdier, 2001, or for the case of a continuous trait or preference, Pichler, 2010; Büchel et al., 2012; Panebianco, 2013; Gravilets, 2021).

2.3 Dynamic optimization problem of the manager

The manager's objective is to steer the organization toward a desired distribution of preferences by allocating effort levels across workers, taking into account both their current preferences and the costs of exerting effort. In essence, the manager aims to promote behaviors that align with the company's objectives, values, and strategies However, under an incomplete contract framework, the manager cannot directly control workers' actions in each period. Instead, influence is exerted dynamically by shaping preferences over time, thereby indirectly guiding behavior. For each worker i, the manager considers such a target as $\bar{S}_i \in [0, 1]$. The manager will then decide levels of effort for each worker i as strategic interventions designed to influence and shape workers' behavior in ways that contribute to organizational success. With a focus on long-term planning, the manager makes his efforts more purposeful and less short-sighted; consequently, from the manager's viewpoint, this scenario presents an optimal control challenge. This approach considers the manager selecting a temporal sequence of effort levels, denoted as e_i^t for each member of the organization.

The cost function is contingent upon not only the level of effort but also the "cultural distance" between the manager and the workers. This cultural distance in period t is quantified as the difference between the manager's preference for worker i, denoted as $\bar{S}_i$, and the worker's interim preference $\hat{S}_i^t$. The greater the "cultural distance," the more expensive an additional unit of effort invested in worker i becomes. Therefore, we represent the following indirect cost function as:

$$C(e_i^t, \bar{S}_i, \hat{S}_i^t) = \frac{\beta}{2}(e_i^t)^2(\bar{S}_i - \hat{S}_i^t)^2, \qquad i = 1, \ldots, N, \tag{4}$$

with $\beta > 0$.

The instantaneous utility function of the manager at stage t, for decisions of investment $e^t = (e_1^t, \ldots, e_N^t)$, contains two parts: the first part, $(\bar{S}_i - S_i^{t+1})^2$, corresponds to the reduction of the distance of the target preference of each worker $\bar{S}_i$

and the changed preference of worker i for the next period S_i^{t+1}. The second part, $\frac{\beta}{2}(e_i^t)^2(\bar{S}_i - \hat{S}_i^t)^2$, corresponds to the cost of exerting e_i^t. Formally:

$$U^t(S^t, S^{t+1}) = -\sum_{i=1}^{N}\left[(\bar{S}_i - S_i^{t+1})^2 + \frac{\beta}{2}(e_i^t)^2(\bar{S}_i - \hat{S}_i^t)^2\right]. \tag{5}$$

By considering the dynamic process, the decision of the manager corresponds to the following dynamic programming problem.

Problem 1 *: Given a manager's ideal vector of preferences $\bar{S}$, an initial condition S^0, and a discount factor $\delta \in (0,1)$, determine the sequence of effort levels $\{e^t\}_{t=0}^{\infty}$ that minimizes*

$$\sum_{t=0}^{\infty}\delta^t\left\{-\sum_{i=1}^{N}\left[(\bar{S}_i - S_i^{t+1})^2 + \frac{\beta}{2}(e_i^t)^2(\bar{S}_i - \hat{S}_i^t)^2\right]\right\}, \tag{6}$$

subject to the conditions

$$e_i^t \in [0,1], \tag{7a}$$

$$\hat{S}_i^t = \gamma x_i^t + (1-\gamma)S_i^t, \tag{7b}$$

$$S_i^{t+1} = e_i^t\bar{S}_i + (1-e_i^t)\hat{S}_i^t, \qquad i = 1,\ldots,N, \quad t \geq 0, \tag{7c}$$

and a vector x^t corresponding to the Nash equilibrium of the stage game.

3 The dynamics of corporate culture: the optimal policy

In this section we characterize the optimal policy of a manager addressing the mathematical problem outlined earlier. We start by computing the Nash equilibrium corresponding to the stage game played by the workers and we ask ourselves how the culture of a group of workers would be like without the presence of any manager. Subsequently, we tackle the dynamic optimization problem for managers to derive the expression for the optimal policy.

3.1 The Nash equilibrium of the workers

Workers' decisions depict a scenario with two temporal dimensions. The first aspect involves an N-player game under complete information, where their utility function is shaped by both their inherent preferences and the prevailing group culture. Let us consider that the N-player game is played by the workers under complete information and assume that in each period there is immediate behavioral adjustment that maintains equilibrium play. Therefore, at period t workers play their the Nash equilibrium action denoted by $\hat{x}_i^t$. Proposition 1 states the unique pure strategy Nash equilibrium of the above game.

Proposition 1 *Let $t > 0$. There exists a unique pure strategy Nash equilibrium $\hat{x}^t = (\hat{x}_1^t, \ldots, \hat{x}_N^t)$ of the stage game given by:*

$$\hat{x}_i^t = \frac{S_i^t + \tilde{\omega}\,\langle S^t \rangle}{1+\tilde{\omega}}, \qquad i = 1, \ldots, N, \tag{8}$$

where we define $\tilde{\omega} = \omega\left(1 - \frac{1}{N}\right)$.

Appendix 1 presents the corresponding proof.

The Nash equilibrium action $\hat{x}_i^t$ of each worker i at each period t is a convex combination of her preference S_i^t and the average $\langle S^t \rangle$ of the preferences of all members in the organization. Recall that we assume that each worker's impact on the group average action $\langle x^t \rangle$ is negligible, workers perceive $\langle x^t \rangle$ as externally given. The relative relevance of each contribution depends on the level of conformism of the worker.

By replacing in Eq.(2) x_i^t by the Nash equilibrium value given by Eq.(8), we obtain

$$\hat{S}_i^t = S_i^t + \tilde{\gamma}[\langle S^t \rangle - S_i^t], \qquad i = 1, \ldots, N, \tag{9}$$

where $\tilde{\gamma} = \dfrac{\gamma\tilde{\omega}}{1+\tilde{\omega}}$.

3.2 The absence of the manager: a benchmark

Before we analyze the long run corporate culture, it would be useful to study as a benchmark case, a scenario where there is no manager in the organization. This analysis would help us to better explain in the sections that follow the profound effect of the manager on the preference dynamic of workers and the ultimate corporate culture.

We begin by assuming that only consistency matters for the evolution of preferences. Thus, at stage t, every worker i plays her Nash equilibrium action and updates her preference S_i^t reducing the dissonance between her current action and her preference, namely $S_i^{t+1} = \hat{S}_i^t$, or

$$S_i^{t+1} = S_i^t + \tilde{\gamma}[\langle S^t \rangle - S_i^t], \qquad i = 1, \ldots, N. \tag{10}$$

The next proposition states the steady state when all members of the organization show strictly positive levels of consistency and conformity.

Proposition 2 *Assume that for all workers $\omega > 0$ and $\gamma > 0$, then in the absence of a manager, the organization tends to complete homogeneity determined by the distribution of initial preferences. The steady state is common for all workers and equals the average initial preferences of the organization*

$$S_i^{\text{st}} = \left\langle S^0 \right\rangle, \qquad i = 1, \ldots, N. \tag{11}$$

Moreover, the organization is fully homogeneous not only in preferences but also in behavior i.e., $\hat{x}_i^{\text{st}} = \langle S^0 \rangle, \forall i$.

Appendix 1 presents the corresponding proof.

Note that both conformism and consistency contribute to creating a homogeneous organization. Conformism drives worker's behavior toward the group's average action, which coincides with a weighted average of the preferences of the members of the group[9]. Similarly, consistency causes workers to adjust their preferences towards their equilibrium action. However, a worker only changes her preference when she displays positive levels of consistency, moving it towards her equilibrium action. Thus, the combination of conformity and consistency explains two fundamental features of an organization without a manager in the long run: homogeneity and dependence on the initial distribution of preferences. Previous research on organizations has shown a strong tendency towards homogeneity (Carrillo and Gromb, 1999; Carrillo and Gromb, 2007) as well as the relevance of hysteresis, i.e., the impact of initial historical conditions. Our first result provides microfoundations for these findings.

3.3 Manager optimal dynamic policy

In this subsection, we present the maximization problem that the manager faces to instill his optimal corporate culture designed as the vector $\{\bar{S}_i\}$. Our analysis centers on solving Problem (1), addressing its existence through dynamic programming techniques. Before this, let us write the problem taking into account the formal expression of the Nash equilibrium behavior.

First, we note that according to the law of motion of the dynamics (3) and Eq. (9), the cost function can be expressed as $C(e_i^t, S_i^t, S_i^{t+1}) = \frac{\beta}{2}\left(S_i^{t+1} - \hat{S}_i^t\right)^2 =$

[9] This average is kept constant throughout the organization's trajectory.

$\frac{\beta}{2}(S_i^{t+1} - S_i^t - \tilde{\gamma}(\langle S^t \rangle - S_i^t))^2$. Therefore, the instantaneous payoff function of the manager becomes:

$$U^t(S^t, S^{t+1}) = -\sum_{i=1}^{N} \left[(\bar{S}_i - S_i^{t+1})^2 + \frac{\beta}{2}(S_i^{t+1} - S_i^t - \tilde{\gamma}(\langle S^t \rangle - S_i^t))^2 \right], \tag{12}$$

Problem 1 is equivalent to the following one by making a change of control variable from e_i^t to S_i^{t+1}.

Problem 2 : *Given a manager's ideal vector of preferences $\bar{S}$, an initial condition S^0, and a discount factor $\delta \in (0,1)$, determine the sequence $\{S^{t+1}\}_{t=0}^{\infty}$ that minimizes*

$$\sum_{t=0}^{\infty} \delta^t \left\{ -\sum_{i=1}^{N} \left[(\bar{S}_i - S_i^{t+1})^2 + \frac{\beta}{2}(S_i^{t+1} - S_i^t - \tilde{\gamma}(\langle S^t \rangle - S_i^t))^2 \right] \right\} \tag{13}$$

subject to the conditions

$$S_i^{t+1} \in [\hat{S}_i^t, \bar{S}_i], \quad \text{if } \hat{S}_i^t \leq \bar{S}_i, \tag{14}$$

$$S_i^{t+1} \in [\bar{S}_i, \hat{S}_i^t], \quad \text{if } \hat{S}_i^t > \bar{S}_i, \qquad i = 1, \ldots, N, \tag{15}$$

with $\hat{S}_i^t = S_i^t + \tilde{\gamma}(\langle S^t \rangle - S_i^t)$

The constraints of Problem (2) define a correspondence $G : \mathbf{Z} \times [0,1]^N \rightrightarrows [0,1]^N$, such that S^{t+1} belongs to $G(t, S^t)$. Moreover, the properties of the instantaneous utility function of the manager, Eq. (12), and the constraint correspondence G are characterized in Lemma 2 in Appendix 1. These properties allow us to tackle the Problem (2) by using the Bellman equation associated with the dynamic programming problem. Formally:

$$V(S^t) = \max_{S^{t+1}} \left\{ U^t(S^t, S^{t+1}) + \delta \cdot V(S^{t+1}) \right\}_{\{t=0,1\ldots\}} \tag{16}$$

and $S^{t+1} = h(S^t)$.

The objective is to obtain the optimal policy function $h(S^t)$ and the value function $V(\cdot)$ that solve the previous functional equation. The recurrence relation $S^{t+1} = h(S^t)$, starting from S^0 provides a solution of Problem (2).

Given the properties of the instantaneous utility function $U(S^t, S^{t+1})$ and the constraint correspondence from Lemma 2, we know by standard arguments[10] that two functions exist, $V(S^t)$ and $h(S^t)$, such that $V(S^t)$ is uniquely defined, continuous, concave, and differentiable which characterizes the solution of Problem (2).

[10] See for instance Theorem 6.10 in Acemoglu, 2009.

Moreover, $h(S^t)$ is single-valued. The optimal plan $h(S^t)$ must satisfy the following set of necessary and sufficient conditions whenever S_i^{t+1} are interior.

The first order condition with respect to S_i^{t+1} establishes that;

$$2(\bar{S}_i - S_i^{t+1}) - \beta(S_i^{t+1} - S_i^t - \tilde{\gamma}(\langle S^t \rangle - S_i^t) + \delta\frac{\partial V(S^{t+1})}{\partial S_i^{t+1}} = 0, \qquad i = 1, \ldots, N. \quad (17)$$

Since $V(S^t)$ is differentiable, we can apply the envelope theorem and differentiate with respect to S_i^t, yielding:

$$\frac{\partial V(S^t)}{\partial S_i^t} = \beta\alpha(S_i^{t+1} - S_i^t - \tilde{\gamma}(\langle S^t \rangle - S_i^t)), \qquad i = 1, \ldots, N, \quad (18)$$

where $\alpha = 1 - \tilde{\gamma}(1 - \frac{1}{N})$. Therefore, we have the following set of Euler equations which solve for $h(S^t)$ as the optimal socialization function:

$$\beta(h_i(S^t) - S_i^t - \tilde{\gamma}[\langle S^t \rangle - S_i^t]) - 2(\bar{S}_i - h_i(S^t)) = \delta\alpha\beta(h_i(h(S^t)) - h_i(S^t) - \tilde{\gamma}[\langle h(S^t) \rangle - h_i(S^t)]$$

Replacing $S_i^{t+1} = h_i(S^t)$ and reordering we get a set of coupled second-order difference equations:

$$S_i^{t+2} = \left(1 - \tilde{\gamma} + \frac{2+\beta}{\beta\alpha\delta}\right) S_i^{t+1} + \tilde{\gamma}\langle S^{t+1} \rangle - \frac{1-\tilde{\gamma}}{\alpha\delta} S_i^t - \frac{\tilde{\gamma}}{\alpha\delta}\langle S^t \rangle - \frac{2}{\alpha\beta\delta}\bar{S}_i, \quad (19)$$

for $i = 1, \ldots, N$.

The above set of coupled second-order difference equations, together with the vector of initial conditions $(S_1^0, \ldots, S_N^0)$ and the transversality condition given by

$$\lim_{t\to\infty} \delta^t \sum_{i=1}^{N} \frac{\partial U(S^t, S^{t+1})}{\partial S_i^t} \cdot S_i^t = 0, \quad (20)$$

serving as boundary conditions, allows us to obtain the solution, in fact, optimal (see Appendix 1, subsection 7).

The next proposition characterizes the optimal policy function that solves this problem and the optimal trajectory of the workers' preference $\{S_i^t\}_{i=1,\ldots,N}$ and shows its convergence.

Proposition 3 *: The optimal socialization plan $S^{t+1} = h(S^t)$ is given by the set of recurrence relations:*

$$S_i^{t+1} = S_i^{\mathrm{st}} + \lambda_2(S_i^t - S_i^{\mathrm{st}}) + (\lambda_1 - \lambda_2)(\langle S^t \rangle - \langle S^{\mathrm{st}} \rangle), \qquad i = 1, \ldots, N \quad (21)$$

where S_i^{st} is the steady state preference of worker i, and $|\lambda_k| < 1$ for $k = 1, 2$.

For any initial condition S^0, the optimal trajectory of the organization $\{S_i^t\}$ and the optimal socialization plan $\{e_i^t\}$ of the manager for $i = 1, \ldots, N$ are given by[11]*:*

$$S_i^t = S_i^{\text{st}} + \lambda_2^t(S_i^0 - S_i^{\text{st}}) + (\lambda_1^t - \lambda_2^t)(\langle S^0 \rangle - \langle S^{\text{st}} \rangle). \tag{22}$$

Appendix 1 presents the corresponding proof and provides explicit expressions for λ_1, λ_2 and S^{st}.

3.4 The steady state corporate culture

The above subsection 3.3 offers a characterization of the manager's policy, shedding light on the preference motion law for each worker. The above results together with the Euler's equations, in particular the set of coupled second order difference equations Eq. (19), allow us to compute the steady-state solution of the organization. The following Proposition 4 presents the expression of the steady state that a forward-looking manager achieves when workers exhibit positive levels of conformity and consistency. This expression yields a long-term preference for a worker i, which is a convex combination of the manager's target $\bar{S}_i$ and the corporation's average target $\langle \bar{S} \rangle$. Such characterization facilitates the understanding of the effect of the manager's target constrained by the optimization parameters such as the discount factor δ and the parameter β that determines the cost of cultural distance between the manager's target for worker i and the preference of this worker. Furthermore, it allows us to explore the impact of the workers' deep culture on the eventual configuration of the company materialized as the conformism and consistency levels, ω and γ, respectively. This particular study is part of the subsection 6 where we develop the impact of the deep culture and its implications. Consequently, both the manager's and the workers' characteristics contribute to shaping the final corporate culture.

Proposition 4 *Assume that $\omega > 0$ and $\gamma > 0$.*

The preference in the long run of each member of the organization with a forward-looking manager is a convex combination of the target of the manager for this worker $\bar{S}_i$ and the average target for the whole organization $\langle \bar{S} \rangle$. Specifically, the steady state of the dynamics is given by:

$$S_i^{\text{st}} = \tilde{d}\bar{S}_i + (1 - \tilde{d}) \langle \bar{S} \rangle, \qquad i = 1, \ldots, N, \tag{23}$$

with $\tilde{d} = \dfrac{2}{2 + \beta\tilde{\gamma}(1 - \alpha\delta)}$.

[11] Note that, being λ_1 and λ_2 real numbers, the notation λ_k^t means the t−th power of λ_k, $k = 1, 2$.

Appendix 1 presents the corresponding proof.

The first finding from proposition 4 enables us to assess the manager's influence relative to the initial distribution of workers' preferences. In subsection 3.2, we looked at the steady-state of the corporate culture in the absence of a manager. Now, we see that the steady-state outcome remains independent of the initial condition of the organization S_i^0. This is clearly in contrast to organizations without a manager where managerial strategies are contingent upon initial conditions.

Another significant finding from the above proposition is that each worker maintains a distance from the average that is a constant proportion of the targeted distance. This is so because the ratio of the distance between $S_i^{\rm st}$ and $\bar{S}_i$ to the distance between $\bar{S}_i$ and the average $\langle \bar{S} \rangle$ remains constant for each i. Specifically, $(S_i^{\rm st} - \bar{S}_i) = (1 - \tilde{d})(\langle \bar{S} \rangle - \bar{S}_i)$ for all i. This finding tells us that the difference between what each worker achieves and their personal target is always a certain fraction of the difference between the organization average and the manager's target. In simple terms, if a worker's objective i is far from what is considered normal for the company, on average, it is harder for the manager to influence them to meet his target. Therefore, the manager will have less success in shaping the worker's preferences.

In this context, the parameter $\tilde{d}$ captures the distance in steady state between the target for worker i and the average of the whole corporation. This distance is influenced by some manager's parameters, such as the discount factor δ and the cost parameter β. By simple computation $\frac{\partial \tilde{d}}{\partial \delta} = \frac{2\beta\tilde{\gamma}\alpha}{(2+\beta\tilde{\gamma}(1-\alpha\delta))^2} > 0$ and $\frac{\partial \tilde{d}}{\partial \beta} = \frac{-2\tilde{\gamma}(1-\alpha\delta)}{(2+\beta\tilde{\gamma}(1-\alpha\delta))^2} < 0$. Therefore, the higher the discount factor δ of the manager and the lower the marginal cost of socialization influenced by parameter β, the closer the steady state $S_i^{\rm st}$ is to the target $\bar{S}_i$. Moreover, the distance $\tilde{d}$ is also marked by the deep culture among workers, captured by expression $\tilde{\gamma}$ which depends on the levels of consistency γ and of conformism ω. This impact is analyzed in Section 6.

We end this subsection by analyzing how much the manager should invest on changing the preference of each member of the team to get his desired corporate culture materialized in $\bar{S}$. Corollary 1 states the optimal stream of efforts corresponding to worker i that depends on both the initial preference of player i and the target $\bar{S}_i$. This expression comes from Eq. (22) with the dynamic Eq. (3).

Corollary 1 *For any initial condition S^0, the optimal socialization plan $\{e_i^t\}$ of the manager for $i = 1, \ldots, N$ is given by:*

$$e_i^t = \frac{\tilde{\gamma}[S_i^{\rm st} - \langle S^{\rm st} \rangle] - \lambda_2^t(1 - \tilde{\gamma} + \lambda_2)[S_i^0 - S_i^{\rm st}] - (\lambda_1^{t+1} - \lambda_2^{t+1})(\lambda_1 - \lambda_2)(1 - \tilde{\gamma})[\langle S^0 \rangle - \langle S^{\rm st} \rangle]}{(\bar{S}_i - S_i^{\rm st}) + \tilde{\gamma}[S_i^{\rm st} - \langle S^{\rm st} \rangle] - (1 - \tilde{\gamma})(\lambda_2^t(S_i^0 - S_i^{\rm st}) + (\lambda_1^t - \lambda_2^t)[\langle S^0 \rangle - \langle S^{\rm st} \rangle)]}. \tag{24}$$

Appendix 1 presents the corresponding proof.

4 The success of the manager

We are interested in measuring how much the manager achieves his objective, in essence, his degree of success. We can say that a manager achieves full success when he establishes his optimal corporate culture, which is indicated by workers reaching S_i^{st} that matches $\bar{S}_i$. This is equivalent to minimizing the aggregate distance between the manager's targets for the workers and their steady state preferences. This achievement could be analyzed through the expression $\Pi^{st} = -\frac{1}{N}\sum_{i=1}^{N}(\bar{S}_i - S_i^{\text{st}})^2$, which we call "success of the manager". A higher value Π^{st} indicates that the steady state composition of the group falls far short of the desired ideal composition for the manager, therefore obtaining a lower level of success.

Proposition 5 *Denote by* $\sigma^2[\bar{S}] = \frac{1}{N}\sum_{i=1}^{N}(\bar{S}_i - \langle\bar{S}\rangle)^2$ *the variance of the manager's preference for the organization. The success of the manager in the steady state is given by:*

$$\Pi^{\text{st}} = -N(1-\tilde{d})^2\sigma^2[\bar{S}]. \tag{25}$$

The proof of Proposition 5 is straightforward by simple computations. Through algebraic manipulation, Π^{st} can be simplified to $-N(1-\tilde{d})^2\sigma^2[\bar{S}]$, since $(\bar{S}_i - S_i^{st}))^2 = (\bar{S}_i - (\tilde{d}\bar{S}_i - (1-\tilde{d})\langle\bar{S}\rangle)^2 = (1-\tilde{d})^2(\bar{S}_i - \langle\bar{S}\rangle)^2$.

It is important to note that the manager attains success on average, as the average steady-state preference aligns with the average target, denoted as $\langle S^{\text{st}}\rangle = \langle\bar{S}\rangle$.

It is easy to examine the impact of the manager's target captured by $\sigma^2[\bar{S}]$ the variance of the manager preference distribution targeted for the group. First of all, achieving homogeneity is a manageable task for the manager. In such cases, the manager achieves full success as the variance of the manager's preference for the organization is zero, consequently Π^{st} is also zero. On the contrary, when the manager's targets for the organization are more heterogeneous, Π^{st} increases proportionally with $\sigma^2[\bar{S}]$. Therefore, the desired heterogeneity from the point of view of the manager has a strong impact on his limits. The manager's success decreases with the dispersion of his targets for the organization. Therefore, there is a genuine trade-off between the ideal targets of the manager for a group and what he can really obtain. In other words, if the manager wishes to obtain some level of success (some average distance of the organization to his targets), then he has to give up some heterogeneity or diversity in his originally preferred goals. It is important to note that $\tilde{d}$ plays a role in $\Pi^{\rm st}$ through the factor $(1-\tilde{d})^2$. The same understanding of the parameters δ and β, and the parameters related to the deep culture of the organization, ω, and γ, applies here as we have discussed earlier regarding the difference between workers' performance and the manager's objective. This implies that if the manager aims for high heterogeneity within the company, their success will likely diminish unless they are prepared to invest more effort.

An interesting exercise would be to compare this level of heterogeneity in the target of the manager with the resulting level of heterogeneity of the organization in the long run. To measure this discrepancy, we compare the variance of the preferences in the steady state, $\sigma^2[S^{\rm st}] = \frac{1}{N}\sum_{i=1}^{N}(S_i^{\rm st} - \langle S^{\rm st}\rangle)^2$ with the variance of the targets of the manager $\sigma^2[\bar{S}]$. Next Corollary 2 relates the targeted heterogeneity with long run heterogeneity both in preferences and behavior.

Corollary 2 *For any given manager's target with variance $\sigma^2[\bar{S}] > 0$, the organization always ends up less heterogeneous in preferences than what the manager desires and also less heterogeneous in behavior than in preferences.*

Proof:

Note that the average steady-state preference coincides with the average target: $\langle S^{\rm st}\rangle = \langle \bar{S}\rangle$. By substituting for the expression of $S_i^{\rm st}$ from Proposition 4 we obtain $\sigma^2[S^{\rm st}] = \tilde{d}^{\,2}\sigma^2[\bar{S}]$. As $\tilde{d} < 1$, it is $\sigma^2[S^{\rm st}] \leq \sigma^2[\bar{S}]$. Thus, the variance of the preferences in the steady state is always smaller than the variance of the desired composition of the organization by the manager.

Let us now consider the variance of the Nash equilibrium actions in the steady state:

$$\sigma^2[\hat{x}^{\rm st}] = \frac{1}{N}\sum_{i=1}^{N}(\hat{x}_i^{\rm st} - \langle \hat{x}^{\rm st}\rangle)^2. \tag{26}$$

After substituting $\hat{x}_i^{\rm st}$ by its value we get $\sigma^2[\hat{x}^{\rm st}] = \left(\frac{\tilde{d}}{1+\tilde{\omega}}\right)^2 \sigma^2[\bar{S}]$. Since $\left(\frac{\tilde{d}}{1+\tilde{\omega}}\right)^2 < \tilde{d}^2$ then $\sigma^2[\hat{x}^{\rm st}] < \sigma^2[S^{\rm st}] < \sigma^2[\bar{S}]$. ■

Consequently, the distance between the actions of the workers and what the manager desires is even greater than the distance from the workers' preferences. Consequently, the manager's success is even more limited when it is measured with respect to behavior.

5 Maintaining the steady-state corporate culture

An important question that arises is whether once the steady-state corporate culture is established, the manager should invest more efforts to maintain it. This question depends on whether the manager aims for either a homogeneous or heterogeneous corporate culture. The impact is significant; in a homogeneous corporate culture, once the company reaches the steady state, the manager does not need to invest any further effort, whereas in a more diverse structure, the manager must invest continuously. Let us denote by e_i^{st} the level of effort per period for each worker i in the steady state. Next Proposition 6 presents the above statement.

Proposition 6

- *If the manager pursues some degree of diversity,* $\sigma(\bar{S}) > 0$*, then* $e_i^{\text{st}} = e^{\text{st}} = \frac{2}{2+\beta(1-\alpha\delta)}$ *for all* i *such that* $[\bar{S}_i - \langle \bar{S} \rangle] \neq 0$.
- *If the manager pursues a completely homogeneous corporate culture,* $\sigma(\bar{S}) = 0$*, then* $e_i^{\text{st}} = e^{\text{st}} = 0$ *for all* i.

Proof

Suppose that the manager does not invest in socialization in a period once workers' preferences correspond to the steady state. Then, the preference of each worker i, according to Eq. (3), moves towards $\hat{S}_i^{\text{st}} = S_i^{\text{st}} + \tilde{\gamma}[\langle \bar{S} \rangle - S_i^{\text{st}}]$.

Notice that $(\hat{S}_i^{\text{st}} - \langle \bar{S} \rangle) = (1-\tilde{\gamma})\tilde{d}(\bar{S}_i - \langle \bar{S} \rangle)$ for all i, while in the steady state this distance was $(S_i^{\text{st}} - \langle \bar{S} \rangle) = \tilde{d}(\bar{S}_i - \langle \bar{S} \rangle)$ for all i. This implies that without the action of the manager, all worker move towards this average $\langle \bar{S} \rangle$ resulting in an organization less and less heterogeneous. Let us compute the level of effort needed to keep each worker in a steady state. Following Eq. (3), $S_i^{\text{st}} = e_i^{\text{st}} \bar{S}_i + (1 - e_i^{\text{st}})\hat{S}_i^{\text{st}}$ and solving by e_i^{st}, we get $e_i^{\text{st}} = \frac{S_i^{st} - \hat{S}_i^{st}}{\bar{S}_i - \hat{S}_i^{st}} = \frac{S_i^{\text{st}} - S_i^{\text{st}} - \tilde{\gamma}[\langle \bar{S} \rangle - S_i^{\text{st}}]}{\bar{S}_i - S_i^{\text{st}} - \tilde{\gamma}[\langle \bar{S} \rangle - S_i^{\text{st}}]} = \frac{\tilde{\gamma}\tilde{d}[\bar{S}_i - \langle \bar{S} \rangle]}{(1-\tilde{d})[\bar{S}_i - \langle \bar{S} \rangle] + \tilde{\gamma}\tilde{d}[\bar{S}_i - \langle \bar{S} \rangle]}$.

Assuming $\sigma(\bar{S}) > 0$, then for those i such that $[\bar{S}_i - \langle \bar{S} \rangle] \neq 0$ then $e_i^{\text{st}} = \frac{\tilde{\gamma}\tilde{d}}{(1-\tilde{d})+\tilde{\gamma}\tilde{d}}$. By including the expression of $\tilde{d}$, this gives us that in the steady state $e_i^{\text{st}} = \frac{2}{2+\beta(1-\alpha\delta)}$.

Assuming now that $\sigma(\bar{S}) = 0$ then $\bar{S}_i = \langle \bar{S} \rangle$ for all i. Therefore $S_i^{\text{st}} = \langle \bar{S} \rangle$ and extra $\tilde{d}$ is not needed. ■

As the value of e_i^{st} does not depend on any specific parameter of the worker i, the manager has to invest at each period and with each worker, a constant and identical amount of effort to maintain the organization in the steady-state configuration S^{st}. This amount is independent of the level of desired diversity of the leader $\sigma(\bar{S})$. Note that this effort level per capita increases with the discount factor δ and decreases with the cost parameter β and the size of the organization N. Paradoxically, it also decreases with higher levels of consistency γ and conformism ω among workers. We discuss the latter result in the next section.

If the manager, in the steady state, does not make a continuous effort to maintain his objective for each worker, the worker's preference worker i would change in that period to $\hat{S}_i^{\mathrm{st}} = S_i^{\mathrm{st}} + \tilde{\gamma}(\langle \bar{S} \rangle - S_i^{\mathrm{st}})$. Therefore, we can measure the cost of keeping the steady state of each worker which should depend on both, e^{st} and the difference between the target $\bar{S}_i$ and $\hat{S}_i^{\mathrm{st}}$ in the following way: $\frac{\beta}{2}(e^{\mathrm{st}})^2(\bar{S}_i - \hat{S}_i^{\mathrm{st}})^2$. By considering all workers we compute the aggregate cost per period needed to sustain the long-run corporate culture. The following Corollary 3 states this aggregate cost:

Corollary 3 *Let $\tilde{d} = \frac{2}{2+\beta\tilde{\gamma}(1-\alpha\delta)}$. The aggregate cost of socialization per period in the steady state is:*

$$C^{st} = \frac{\beta}{2}\tilde{\gamma}^2\tilde{d}^2 N\sigma^2[\bar{S}].$$

Proof Notice that $\bar{S}_i - \hat{S}_i^{\mathrm{st}} = (\tilde{d}\tilde{\gamma} + 1 - \tilde{d})[\bar{S}_i - \langle \bar{S} \rangle]$ by simple but tedious computation. We have defined the total cost as $C^{st} = \frac{\beta}{2}(e^{\mathrm{st}})^2 \sum_{i=1}^{N}(\bar{S}_i - \hat{S}_i^{\mathrm{st}})^2$. Hence,

$$\begin{aligned}
C^{st} &= \frac{\beta}{2}(e^{\mathrm{st}})^2 \sum_{i=1}^{N}(\tilde{d}\tilde{\gamma} + 1 - \tilde{d})^2[\bar{S}_i - \langle \bar{S} \rangle]^2 \\
&= \frac{\beta}{2}(e^{\mathrm{st}})^2(\tilde{d}\tilde{\gamma} + 1 - \tilde{d})^2 \sum_{i=1}^{N}[\bar{S}_i - \langle \bar{S} \rangle]^2 \\
&= \frac{\beta}{2}(e^{\mathrm{st}})^2 \left(\frac{\tilde{\gamma}\tilde{d}}{e^{st}}\right)^2 N\sigma^2[\bar{S}] \\
&= \frac{\beta}{2}\tilde{\gamma}^2\tilde{d}^2 N\sigma^2[\bar{S}]
\end{aligned}$$

■

The manager's targets for the organization, $\sigma^2[\bar{S}]$, have the same impact on the aggregate cost as the success of the manager: a more heterogeneous targeted corporate culture implies higher cost since it increases proportionally with $\sigma^2[\bar{S}]$. The computation of the partial derivative of $\tilde{d}^2$ with respect to β and δ are negative and positive, respectively if $2 - \beta\tilde{\gamma}(1-\alpha\delta) > 0$. Consequently, the more costly the cultural distance is for the manager, the higher the aggregate cost will be. Similarly, the more patience the manager has, δ goes to 1, then the lower the aggregate cost. The influence of the deep culture shared by the workers will be analyzed in the next section.

6 The impact of workers' deep culture on corporate culture

We have focused on the impact of the manager's target or mission, captured by his desired level of heterogeneity for the organization, and on the impact of the parameters that affect his dynamic optimization, in the long run, steady state, corporate culture, and also on the levels of per capita socialization efforts and its aggregate cost, needed to maintain the long run corporate culture. However, an important determinant of the results, as described earlier, are the levels of conformism and consistency shared by the members of the group which constitute their deep culture.

A deep culture with significant enough levels of conformity and/or consistency displayed and shared by the workers generates a strong tendency in the individual's dynamics of her preference towards the average preference of the group. Conformity creates an obvious direct effect in this direction but notice that consistency, the tendency to update preferences in the direction of actions, reinforces indirectly this tendency. Even if a worker is not highly conformist, her preference can still be significantly influenced by the average preference due to the way the Nash equilibrium is determined. Therefore, while conformity directly reinforces homogeneity, consistency does it indirectly by reinforcing the tendency towards the Nash equilibrium. This strong influence of the average group behavior can be denoted, as it is usually done in the literature, as the peer effect. Therefore, a deep culture characterized by high levels of conformity and consistency will result in strong peer effects. The expression $\tilde{\gamma}$, which depends positively on the parameters ω and γ, captures these effects in our model and results.

Next Lemma 1 shows the effects of changes in the parameters ω and γ on the distance $\tilde{d}$ and on the manager's per capita effort in steady state e^{st}.

Lemma 1 *Let* $\gamma \in [0, 1)$ *and* $\omega > 0$.

- $\frac{\partial \tilde{d}}{\partial \gamma} < 0$ *and* $\frac{\partial \tilde{d}}{\partial \omega} < 0$
- $\frac{\partial e^{st}}{\partial \gamma} < 0$ *and* $\frac{\partial e^{st}}{\partial \omega} < 0$

Proof

Notice that $\frac{\partial \tilde{\gamma}}{\partial \gamma} = \frac{\tilde{\omega}}{1+\tilde{\omega}} > 0$, and $\frac{\partial \tilde{\gamma}}{\partial \omega} = \frac{\gamma(1-\frac{1}{N})}{(1+\tilde{\omega})^2} > 0$.

The partial derivative of $\tilde{d}$ in respect of $\tilde{\gamma}$ is $\frac{\partial \tilde{d}}{\partial \tilde{\gamma}} = \frac{-2\beta[(1-\alpha\delta)+\tilde{\gamma}(1-\frac{1}{N})]}{(2+\beta\tilde{\gamma}(1-\alpha\delta))^2} < 0$. Therefore, the first condition holds.

With respect to e^{st} we get $\frac{\partial e^{st}}{\partial \tilde{\gamma}} = \frac{-2\beta\delta(1-\frac{1}{N})}{(2+\beta(1-\alpha\delta))^2} < 0$ giving the second condition. ■

Lemma 1 allows us to interpret the effect of the deep culture on the results presented in the above sections. First of all, in a strong deep culture with significant levels of conformity and consistency the distance between what the manager achieves for each worker i and what he would like to achieve for her, formalized as $(1 - \tilde{d})$ in proposition 4, is high and increases with the levels of conformity and consistency. Consequently, in aggregate terms the manager's success obtained in Proposition 5 is smaller and the heterogeneity of the resulting culture is smaller than the heterogeneity level targeted by the manager and even smaller in terms of equilibrium actions as obtained in corollary 2. However, a weaker deep culture, indicated by lower levels of consistency γ and conformism ω among workers, implies a smaller difference between the steady-state S_i^{st} to the target $\bar{S}_i$. Hence, workers tend to perform closer to their targets. If the level of conformism or the level of consistency tends to zero, in both cases, $\tilde{d}$ tends to 1 and the aggregate success is maximal, i.e., equal to zero. This result occurs because the system of difference equations that governs the dynamics of the group preference distribution becomes uncoupled[12]. The unique force that influences the evolution of individual preferences is the "follow the leader" behavior. An important insight provided by our model is that conformism alone cannot compete or hamper the socialization effort of the manager, neither can consistency alone.

Secondly, Proposition 6 in the previous section, shows that the required level of effort per capita in steady state remains unaffected by the manager's desired level of diversity and is an identical amount of resources for each worker. Note that this effort level per capita increases with the discount factor δ and decreases with the cost parameter β and the size of the organization N. Paradoxically, it also decreases with higher levels of consistency γ and conformism ω among workers. In essence, organizations with a strong workers´ deep culture, that is, with high levels of conformism and/or consistency require less effort per capita from the manager to maintain the resulting corporate culture S^{st}. This is because higher levels of $\tilde{\gamma}$ mean that all organization members are closer to the average ideal action $\langle \bar{S} \rangle$ and further from their individual target $\bar{S}_i$ set by the manager. Consequently, maintaining a less diverse corporate culture is less resource-intensive for the manager in terms of effort per capita. Once again, peer effects play a collaborative role with managerial efforts in maintaining stability within the organization's steady state.

Proposition 6 also sheds light on an extreme consequence of the peer effect within

[12]The system is $S_i^{t+1} = e_i^t \bar{S}_i + (1 - e_i^t) S_i^t$ for all $i = 1, \dots N$, that eventually leads to $S_i^{st} = \bar{S}_i$.

organizational dynamics. Specifically, when the manager aims for a homogeneous corporate culture, the peer effect becomes a valuable asset in maintaining such a structure. Peer effects act as strong allies in sustaining homogeneity within the organization. This is the reason why $e_i^{\mathrm{st}} = 0$, the peer force replaces the manager effort. In the limit, if the manager pursues a completely homogeneous culture, not only does it accomplish this, but it also no longer has to invest in socialization. Conversely, if the manager seeks a degree of diversity, the peer effect works in the opposite direction. If the manager does not exert effort in a given period, each worker i will tend to move closer to the average preference of the organization $\langle \bar{S} \rangle$, leading to a gradual reduction in heterogeneity. As a result, the manager must invest more resources in each worker in every period to uphold the organization's steady-state configuration S^{st}.

Finally, we obtain that the aggregate cost of socialization in the steady state increases with the strength of the deep culture $\tilde{\gamma}$ and the degree of heterogeneity of the manager's target for the organization $\sigma^2[\bar{S}]$ (as shown in Corollary 3). Stronger peer effects, caused by a strong workers´ deep culture, lead to lower percapita socialization efforts in the steady state, as shown previously. However, they also lead to a higher aggregate cost per period. The reason for this is that socialization costs also depend on the cultural distance between the manager's target for each worker and that worker's preference in the steady state. A strong deep culture with significant levels of conformity and consistency leads to greater cultural distance in the steady state. This latter cost-increasing effect dominates the former cost-reducing effect, which is to reduce level of effort so that the final effect is an increase in the aggregate cost per period.

7 Concluding Remarks

We presented a model in which a manager builds corporate culture, recognizing that behavior cannot be imposed through monetary incentives. Instead, the manager invests in costly coaching and social activities to shape team preferences. The manager also acknowledges a deeper culture shaped by shared psychological traits, like conformity and consistency, which influence long-term behaviors and preferences. Our formal model is applied to multi-task team agency relationships, where workers balance multiple tasks, each having their own preference for a particular mix of these tasks.

Our main finding is that when workers' deep culture exhibits positive levels of both conformity and consistency, and the manager seeks a heterogeneous corporate culture, the resulting steady state culture will always be suboptimal, in other words, a "second best." Specifically, the organization will be less diverse than the "first best" culture the manager ideally desires. This impossibility theorem implies that existing heterogeneous corporate cultures are inherently suboptimal. However, our dynamic programming model provides a way to quantify the gap between the actual culture and the manager's target, showing how this gap depends on key model parameters.

We also demonstrate that both conformity and consistency are necessary for this suboptimal outcome. If the workers' deep culture shows only conformity or consistency, the manager can achieve the desired cultural target in the steady state. More generally, these traits have conflicting effects on the manager's success. High levels of conformity or consistency increase the gap between the target and steady-state cultures but simultaneously reduce the per capita investment needed to maintain the steady-state culture.

The model presented in this paper is built upon certain assumptions that align with specific real-world situations. The analysis of other related scenarios is worth further research. We assume that all workers display the same deep culture. This can be considered a reasonable assumption in many long-lived, closed and long-established groups. It would be interesting to analyze a newly established group where there is an heterogeneous distribution of conformity and consistency. Moreover, we have assumed that the manager can influence, at a different cost, in the evolution of the preference between the tasks of each and every member of the group. However, the case of a heterogeneous deep culture and where the manager might not have "parental" influence for some workers is also another future line of research.

A further extension of our approach is related to a change in the assumption we make concerning the effect of conformity on the utility of the workers and, therefore, on their behavior. As is common in much of the literature, we capture the peers' influence in the individual utility function, using the average group behavior. But an important extension is to model the group as a network of relationships where the manager is placed in a pivotal position.

Finally, in this paper, the target of the manager is exogenously given. Further research should address the choice of the manager of a target corporate culture. This would require incorporating production and effort into our model. A key question that deserves further investigation is how the manager might adjust away from his "first-best" corporate culture in anticipation of the group's equilibrium dynamic response, which is shaped by the group's deep culture.

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Appendix 1

Proof of Proposition 1

Given our assumptions on the set of actions x^t and on the payoff functions, the Nash equilibrium $\hat{x}^t$ is computed by solving the set of first-order conditions:

$$\left.\frac{\partial u_i^t(x^t)}{\partial x_i^t}\right|_{x^t=\hat{x}^t} = 0, \qquad i = 1, \ldots, N. \tag{A1}$$

Performing the partial derivatives in (A1) and noting that x_i^t appears in the utility function (1) explicitly as well as in the average $\langle x^t \rangle$, we obtain that the best response function of worker i at time t satisfies:

$$\hat{x}_i^t = \frac{S_i^t + \tilde{\omega} \langle \hat{x}^t \rangle}{1 + \tilde{\omega}}, \qquad i = 1, \ldots, N \tag{A2}$$

where, for brevity in notation, we have defined $\tilde{\omega} = \omega \left(1 - \frac{1}{N}\right)$.

Note that (A2) is not yet the explicit solution for the Nash equilibrium actions $\hat{x}_i^t$ as that expression constitutes actually a set of N coupled equations since $\hat{x}_{j=1,\ldots,N}^t$ appear in the definition of the average $\langle \hat{x}^t \rangle$. The explicit solution is found by taking averages on both sides of this equation:

$$\left\langle \hat{x}^t \right\rangle = \frac{\langle S^t \rangle + \tilde{\omega} \langle \hat{x}^t \rangle}{1 + \tilde{\omega}}, \tag{A3}$$

or

$$\left\langle \hat{x}^t \right\rangle = \left\langle S^t \right\rangle. \tag{A4}$$

Therefore, combining (A2) and (A4), it follows:

$$\hat{x}_i^t = \frac{S_i^t + \tilde{\omega} \langle S^t \rangle}{1 + \tilde{\omega}}, \qquad i = 1, \ldots, N, \tag{A5}$$

i.e., result Eq.(8).

Proof or proposition 2

At the steady state $S_i^t = S_i^{\mathrm{st}}$, $\forall t$, we get from Eq. (10):

$$S_i^{\mathrm{st}} = S_i^{\mathrm{st}} + \tilde{\gamma}[\langle S^{\mathrm{st}} \rangle - S_i^{\mathrm{st}}], \qquad i = 1, \ldots, N. \tag{A6}$$

As $\tilde{\gamma} \neq 0$, we obtain $S_i^{\mathrm{st}} = \langle S^{\mathrm{st}} \rangle$, $\forall i$. Taking averages on both sides of Eq. (10) we arrive in the case $\tilde{\gamma} \neq 0$ of $\langle S^{t+1} \rangle = \langle S^t \rangle$, implying $\langle S^t \rangle = \langle S^0 \rangle$, $\forall t$, extending to the steady state $\langle S^{\mathrm{st}} \rangle = \langle S^0 \rangle$.

We introduce this result in the steady state of the Nash equilibria, Eq. (8):

$$\begin{aligned}\hat{x}_i^{\mathrm{st}} &= \frac{S_i^{\mathrm{st}} + \tilde{\omega}\,\langle S^{\mathrm{st}}\rangle}{1+\tilde{\omega}} \\ &= \frac{\langle S^0\rangle + \tilde{\omega}\langle S^0\rangle}{1+\tilde{\omega}} \\ &= \langle S^0\rangle. \end{aligned} \tag{A7}$$

Properties of the utility function $U(\cdot,\cdot)$ and the constraint correspondence $G(\cdot)$

Lemma 2

(i) *The function $U(S^t, S^{t+1})$ is bounded, continuous and differentiable on the interior of its domain.*

(ii) *The function $U(S^t, S^{t+1})$ is jointly strictly concave.*

(iii) *The constraint correspondence denoted by $G(S^t)$ is non-empty for all $S^t \in X$, compact valued, continuous, convex and monotone.*

Proof of (i)

Note that $U(\cdot,\cdot)$ is the sum of quadratic forms with respect to the state and control variables, then it is clearly continuous, differentiable on the interior of its domain. As S^t and S^{t+1} are defined in the compact set $[0,1]$ then $U(\cdot,\cdot)$ is bounded.

Proof of (ii)

In order to study the concavity condition, we have to compute the Hessian matrix $\mathbf{H}_{2N} \equiv \left[\dfrac{\partial^2 U(S^t,S^{t+1})}{\partial S_i^k \partial S_j^{k'}}\right]_{i,j=1,\ldots,N;k,k'\in\{t,t+1\}}$. After some straightforward calculation we obtain the following block structure for this matrix:

$$\mathbf{H}_{2N} = \left(\begin{array}{c|c} M_N(-\beta\alpha^2, -\beta\alpha\frac{\tilde{\gamma}}{N}) & M_N(\beta\alpha, 0) \\ \hline M_N(\beta\alpha, \beta\frac{\tilde{\gamma}}{N}) & M_N(-(\beta+2), 0) \end{array}\right) \tag{A8}$$

where $\alpha = 1 - \tilde{\gamma}(1-\frac{1}{N})$. Let H_k be the principal minor of size $k \in \{1,\ldots,N,N+1,\ldots,2N\}$ of the Hessian matrix $\mathbf{H}$. In order to prove that the Hessian matrix $\mathbf{H}$ is negative we compute the determinant of all principal minors of $\mathbf{H}$. For $n \in \{1,\ldots,N\}$, from the results in Appendix 2 we obtain the determinant of $H_n = (a-b)^{n-1}(a+(n-1)b)$ where $a = -\beta\alpha^2$ and $b = -\beta\alpha\dfrac{\tilde{\gamma}}{N}$.

Note that $(a + (n-1)b) = -\beta\alpha^2 - (n-1)\beta\alpha\frac{\tilde{\gamma}}{N} < 0$ for all N. Then the sign of $\mathrm{Det}(H_n)$ depends on the sign of $(a-b)^{n-1}$. As $(a-b)^{n-1} = \beta\alpha(\frac{\tilde{\gamma}}{N} - \alpha) = (\tilde{\gamma} - 1)$ then $(a-b)^{n-1} < 0$ when n is odd and positive when n is even.

Consequently, for $k \leq N$ odd, the minor H_k has negative sign and positive sign when k is even.

For $k \in \{N+1, \ldots, 2N\}$ let us proceed by induction. We claim that $\mathrm{Det}(H_k) = (2(1-\beta))^{k-N}\mathrm{Det}(H_N)$ for $2N \geq k > N$. Consider the first case by fixing $k = N+1$. Then the principal minor is:

$$\mathbf{H}_{N+1} \equiv \begin{pmatrix} -\beta\alpha^2 & -\beta\alpha\frac{\tilde{\gamma}}{N} & -\beta\alpha\frac{\tilde{\gamma}}{N} & \ldots\ldots & \beta\alpha \\ -\beta\alpha\frac{\tilde{\gamma}}{N} & -\beta\alpha^2 & -\beta\alpha\frac{\tilde{\gamma}}{N} & \ldots\ldots & 0 \\ \ldots & \ldots & \ldots & \ldots\ldots & \ldots \\ -\beta\alpha & -\beta\frac{\tilde{\gamma}}{N} & -\beta\alpha\frac{\tilde{\gamma}}{N} & \ldots\ldots & -(\beta+2) \end{pmatrix} \tag{A9}$$

By computing the determinant by adjoints we get that $\mathrm{Det}(H_{N+1})$ is equal to $(-1)^{N+2}\beta\alpha(-1)^N\frac{-1}{\alpha}\mathrm{Det}(H_N) + (-1)^{2N+2} - (\beta+2)\mathrm{Det}(H_N) = 2(1-\beta)\mathrm{Det}(H_N)$. Suppose that the claim holds for k. Then $\mathrm{Det}(H_k) = (2(1-\beta))^{k-N}\mathrm{Det}(H_N)$. By making the same computations than the first case, we get the case $k+1$ where $\mathrm{Det}(H_{k+1}) = (-1)^{N+k+1}\beta\alpha(-1)^N\frac{-1}{\alpha}\mathrm{Det}(H_k) + (-1)^{2(N+k)} - (\beta+2)\mathrm{Det}(H_k) = 2(1-\beta)\mathrm{Det}(H_k) = 2(1-\beta)(2(1-\beta))^{k-N}\mathrm{Det}(H_N) = (2(1-\beta))^{k+1-N}\mathrm{Det}(H_N)$.

Therefore the sign changes every stage from the case N. Hence, $U(\cdot,\cdot)$ is strictly concave.

Proof of (iii)

Given the definition of G and the above properties, the result holds.

Proof of Proposition 3

In the proof of this proposition, we will use certain properties—derived in Appendix 2—of a special family of $N \times N$ matrices, denoted by $\mathbf{M}_N(a,b)$, characterized by having a common value a along the diagonal and a common value b in all off-diagonal entries.

Let us define:

$$\mathbf{A}_1 = \mathbf{M}_N(a_1, b_1), \qquad \mathbf{A}_2 = \mathbf{M}_N(a_2, b_2) \tag{A10}$$

with $a_1 = \alpha + \dfrac{2+\beta}{\beta\alpha\delta}$, $b_1 = \dfrac{\gamma}{N}$, $a_2 = -\dfrac{1}{\delta}$, and $b_2 = -\dfrac{\gamma}{\alpha\delta N}$. With this notation the recurrence relation Eq.(19) reads:

$$S^{t+2} = \mathbf{A}_1 S^{t+1} + \mathbf{A}_2 S^t - \frac{2}{\alpha\beta\delta}\bar{S}, \tag{A11}$$

whose stationary value is

$$S^{\rm st} = -\frac{2}{\alpha\beta\delta}\left(\mathbf{I} - \mathbf{A_1} - \mathbf{A_2}\right)^{-1}\bar{S}. \tag{A12}$$

We now convert Eq. (A11) into a homogeneous relation by introducing $Z^t = S^t - S^{\rm st}$. This leads to

$$Z^{t+2} = \mathbf{A}_1\, Z^{t+1} + \mathbf{A}_2\, Z^t. \tag{A13}$$

By trying $Z^t = \mathbf{\Pi}^t C$, being C a vector[13], we derive that the matrix $\mathbf{\Pi}$ must satisfy $\mathbf{\Pi}^2 = \mathbf{A}_1\mathbf{\Pi} + \mathbf{A}_2$. If we now make the ansatz that $\mathbf{\Pi} = \mathbf{M}_N(a,b)$, the two eigenvalues of $\mathbf{\Pi}$, λ_1, λ_2, satisfy quadratic algebraic equations:

$$\lambda_1^2 = \lambda_1(a_1 + b_1(N-1)) + a_2 + b_2(N-1), \tag{A14}$$

$$\lambda_2^2 = \lambda_2(a_1 - b_1) + a_2 - b_2. \tag{A15}$$

The coefficients for both equations are related to the eigenvalues of $\mathbf{A}_1$ and $\mathbf{A}_2$ (see Appendix 2 for details):

$$b_1(N-1) + a_1 = 1 + \frac{\frac{2}{\beta}+1}{\alpha\delta}, \tag{A16a}$$

$$b_2(N-1) + a_2 = -\frac{1}{\alpha\delta}, \tag{A16b}$$

$$a_1 - b_1 = 1 - \gamma + \frac{\frac{2}{\beta}+1}{\alpha\delta}, \tag{A16c}$$

$$a_2 - b_2 = -\frac{1-\gamma}{\alpha\delta}. \tag{A16d}$$

We prove now a lemma:

Lemma:

Let λ^-, λ^+ be the two roots of the equation $\lambda^2 - (a + AB)\lambda + aA = 0$ with $0 < a < 1, A > 1, B > 1$. Then the roots are real and satisfy $0 < \lambda^- < a, \lambda^+ > A$.

Proof:

The roots are:

$$\lambda^\pm = \frac{1}{2}\left[a + AB \pm \sqrt{(a+AB)^2 - 4aA}\right]. \tag{A17}$$

Both roots are real as $(a + AB)^2 > (a + A)^2 > 4aA$, the latter inequality being equivalent to $(A - a)^2 > 0$. As we are subtracting from $a + AB$ a smaller number it

[13] The notation $\mathbf{\Pi}^t$ means the t-power of matrix $\mathbf{\Pi}$.

turns out that $\lambda^- > 0$. We now prove that $\lambda^- < a$ or $AB - a < \sqrt{(a+AB)^2 - 4aA}$. As $AB > a$ we can square safely to prove that $(AB-a)^2 < (a+AB)^2 - 4aA$ or $aA < aAB$ which is true. As the product of the roots is aA and $\lambda^- < a$ it follows that $\lambda^+ > A$.

Introducing $B = 1 + 2/\beta$ and $A = 1/(\alpha\delta)$ and $a = 1$ or $a = 1 - \gamma$ this lemma proves that $0 < \lambda_1^- < 1, \lambda_1^+ > 1/(\alpha\delta) > 1$ and $0 < \lambda_2^- < 1 - \gamma, \lambda_2^+ > 1/(\alpha\delta) > 1$. Explicit expressions are:

$$\lambda_1^\pm = \frac{1}{2}\left(1 + AB \pm \sqrt{(1+AB)^2 - 4A}\right), \tag{A18}$$

$$\lambda_2^\pm = \frac{1}{2}\left(1 - \gamma + AB \pm \sqrt{(1-\gamma+AB)^2 - 4(1-\gamma)A}\right). \tag{A19}$$

We construct $\mathbf{\Pi}_+ = \mathbf{M}_N(a^+, b^+)$ using the two eigenvalues λ_1^+, λ_2^+ larger than 1, and $\mathbf{\Pi}_- = \mathbf{M}_N(a^-, b^-)$ using the two eigenvalues λ_1^-, λ_2^- smaller than 1. Therefore the corresponding values of $a^\pm$ and $b^\pm$ follow from:

$$a^\pm = \frac{\lambda_1^\pm + (N-1)\lambda_2^\pm}{N}, \tag{A20}$$

$$b^\pm = \frac{\lambda_1^\pm - \lambda_2^\pm}{N}. \tag{A21}$$

The solution can be written as

$$Z^t = \mathbf{\Pi}_+^t C_1 + \mathbf{\Pi}_-^t C_2, \tag{A22}$$

or, undoing the change of variables,

$$S^t = S^{\text{st}} + \mathbf{\Pi}_+^t C_1 + \mathbf{\Pi}_-^t C_2, \tag{A23}$$

where vectors C_1 and C_2 are chosen to satisfy the initial and boundary conditions.

As the eigenvalues satisfy $\lambda_{1,2}^+ > 1$, it turns out that the coefficients of $\mathbf{\Pi}_+^t$ diverge for $t \to \infty$. The transversality condition hence requires that the vector C_1 is equal to 0 (see next subsection). Vector C_2 is then obtained by satisfying the initial condition S^0 at $t = 0$, as $C_2 = S^0 - S^{\text{st}}$. Consequently, the solution of the recurrence relation for vector S^t is:

$$S^t = S^{\text{st}} + \mathbf{\Pi}_-^t \left(S^0 - S^{\text{st}}\right). \tag{A24}$$

which, using the properties of matrices $\mathbf{M}_N(a,b)$, can be written in the form of Eq.(22) with $\lambda_k = \lambda_k^-$, $k = 1, 2$, as given by Eqs. (A18,A19).

Transversality condition

Lemma: The condition of transversality holds if and only if the vector of parameters C_1 is the vector 0.

Proof:

The condition of transversality states:

$$\lim_{t\to\infty} \delta^t \sum_{i=1}^{N} \frac{\partial U(S^t, S^{t+1})}{\partial S_i^t} \cdot S_i^t = 0. \tag{A25}$$

We first compute

$$\frac{\partial U(S^t, S^{t+1})}{\partial S_i^t} = \beta(1-\gamma)(S_i^{t+1} - S_i^t - \gamma(\langle S^t\rangle - S_i^t)) + \beta\gamma(\langle S^{t+1}\rangle - \langle S^t\rangle)$$

which, replaced in Eq.A25 leads to

$$\lim_{t\to\infty} \delta^t \left[\gamma\left(\langle S^{t+1}S^t\rangle - \langle S^{t+1}\rangle\langle S^t\rangle\right) + \langle S^t\rangle^2 - \langle S^{t+1}S^t\rangle + (1-\gamma)^2\left(\langle (S^t)^2\rangle - \langle S^t\rangle^2\right)\right] = 0. \tag{A26}$$

Let us recall that $S^t = S^{st} + \mathbf{\Pi}_+^t C_1 + \mathbf{\Pi}_-^t C_2$, we now express the term

$$\left[\gamma\left(\langle S^{t+1}S^t\rangle - \langle S^{t+1}\rangle\langle S^t\rangle\right) + \langle S^t\rangle^2 - \langle S^{t+1}S^t\rangle + (1-\gamma)^2\left(\langle (S^t)^2\rangle - \langle S^t\rangle^2\right)\right]$$

by substituting[14] the expression of S^t. In order to get our result, we extract such summands containing C_1. Notice that any other summand with neither C_1 nor C_2 will converge to 0 when t goes to infinity, since it is constant and it is multiplied by δ^t. The same argument holds for those summands with C_2 because all of them are multiplied by powers of eigenvalues with module smaller than 1.

Let us denote by $f(\gamma, \lambda_{1+}, \lambda_{1-}, \lambda_{2+}, \lambda_{2-})$ the term that contains C_1 in all their summands:

$$\begin{aligned}
&\gamma[[\lambda_{2+}^t(\langle S^{st}C_1\rangle - \langle S^{st}\rangle\langle C_1\rangle) + \lambda_{2+}^{t+1}(\langle S^{st}C_1\rangle - \langle S^{st}\rangle\langle C_1\rangle)\\
&+\lambda_{2+}^{2t+1}(\langle C_1C_1\rangle - \langle C_1\rangle\langle C_1\rangle) + \lambda_{2+}^{t+1}\lambda_{2-}^t(\langle C_1C_2\rangle - \langle C_1\rangle\langle C_2\rangle)\\
&+\lambda_{2+}^t\lambda_{2-}^{t+1}(\langle C_1C_2\rangle - \langle C_1\rangle\langle C_2\rangle)]\\
&+(1-\gamma)^2[\lambda_{2+}^t(\langle S^{st}C_1\rangle - \langle S^{st}\rangle\langle C_1\rangle) + \lambda_{2+}^t\lambda_{2-}^t(\langle C_1C_2\rangle - \langle C_1\rangle\langle C_2\rangle)\\
&+\lambda_{2+}^{2t}(\langle C_1C_1\rangle - \langle C_1\rangle^2)]\\
&+\lambda_{1+}^t\langle S^{st}\rangle\langle C_1\rangle + \lambda_{2+}^t(\langle S^{st}\rangle\langle C_1\rangle - \langle S^{st}C_1\rangle)\\
&+\lambda_{1+}^{t+1}\langle S^{st}\rangle\langle C_1\rangle + \lambda_{2+}^{t+1}(\langle S^{st}C_1\rangle - \langle S^{st}\rangle\langle C_1\rangle)\\
&+(\lambda_{1+}^{2t} - \lambda_{1+}^{2t+1})\langle C_1\rangle^2 + \lambda_{2+}^{2t+1}(\langle C_1\rangle^2 - \langle C_1C_1\rangle)\\
&+2\lambda_{1+}^t\lambda_{1-}^t\langle C_1\rangle\langle C_2\rangle - \lambda_{1+}^{t+1}\lambda_{1-}^t\langle C_1\rangle\langle C_2\rangle - \lambda_{1+}^t\lambda_{1-}^{t+1}\langle C_1\rangle\langle C_2\rangle\\
&+\lambda_{2+}^{t+1}\lambda_{2-}^t(\langle C_1\rangle\langle C_2\rangle - \langle C_1C_2\rangle) + \lambda_{2+}^t\lambda_{2-}^{t+1}(\langle C_1\rangle\langle C_2\rangle - \langle C_1C_2\rangle)
\end{aligned}$$

[14]Computations are available on request.

1. First, let us consider the above expression $f(\gamma, \lambda_{1+}, \lambda_{1-}, \lambda_{2+}, \lambda_{2-})$ as a polynomial in $\mathbb{R}[\gamma, \lambda_{1+}, \lambda_{1-}, \lambda_{2+}, \lambda_{2-}]$. Therefore, the transversality condition is equivalent to saying that the above polynomial must be equal to the polynomial 0 in $\mathbb{R}[\gamma, \lambda_{1+}, \lambda_{1-}, \lambda_{2+}, \lambda_{2-}]$, otherwise the transversality condition does not hold.

2. Note then that all coefficients must be zero, in particular those as $(\langle X \cdot C_1 \rangle - \langle X \rangle \langle C_1 \rangle) = 0$ for all X. Therefore either $C_1 = 0$ or C_1 verifies that $\langle X \cdot C_1 \rangle = \langle X \rangle \langle C_1 \rangle$ for all X. Adding this condition in the expression $f(\gamma, \lambda_{1+}, \lambda_{1-}, \lambda_{2+}, \lambda_{2-})$ we get the following polynomial: that cannot be zero unless $C_1 = 0$.

$$\begin{aligned}&\lambda_{1+}^{t}\langle S^{st}\rangle\langle C_1\rangle + \lambda_{1+}^{t+1}\langle S^{st}\rangle\langle C_1\rangle + (\lambda_{1+}^{2t} - \lambda_{1+}^{2t+1})\langle C_1\rangle^2 + 2\lambda_{1+}^{t}\lambda_{1-}^{t}\langle C_1\rangle\langle C_2\rangle\\ &-\lambda_{1+}^{t+1}\lambda_{1-}^{t}\langle C_1\rangle\langle C_2\rangle - \lambda_{1+}^{t}\lambda_{1-}^{t+1}\langle C_1\rangle\langle C_2\rangle\end{aligned}$$

Given that λ_{1+} and λ_{1-} are independent, in order to get the transversality condition it is necessary that $C_1 = 0$. Consequently, the result holds.

Proof of Proposition 4

Setting in Eq. (19) the steady state $S^t = S^{\text{st}}$, $\forall t$, we arrive after simple algebra to

$$2(\bar{S}_i - S_i^{\text{st}}) + \beta\tilde{\gamma}(1 - \delta\tilde{\gamma})(\langle S^{\text{st}}\rangle - S_i^{\text{st}}) = 0. \tag{A27}$$

Taking averages:

$$2(\langle \bar{S}\rangle - \langle S^{\text{st}}\rangle) + \beta\tilde{\gamma}(1 - \delta\tilde{\gamma})(\langle S^{\text{st}}\rangle - \langle S^{\text{st}}\rangle) = 0 \tag{A28}$$

Therefore $\langle \bar{S}\rangle = \langle S^{\text{st}}\rangle$. By using $\langle \bar{S}\rangle$ in place of $\langle S^{\text{st}}\rangle$ in Eq. (A27) we obtain

$$S_i^{\text{st}} = \frac{2\bar{S}_i + \beta\tilde{\gamma}(1 - \delta\tilde{\gamma})\langle \bar{S}\rangle}{2 + \beta\tilde{\gamma}(1 - \delta\tilde{\gamma})}, \tag{A29}$$

which is equivalent to Eq. (23) in the main text. From this expression it follows that $\langle S^{\text{st}}\rangle = \langle \bar{S}\rangle$.

Proof of Corollary 1

Combining Eqs. (3) and (9) we can express e_i^t as

$$e_i^t = \frac{S_i^{t+1} - S_i^t - \tilde{\gamma}(\langle S^{\text{st}}\rangle - S_i^t)}{\bar{S}_i - S_i^t - \tilde{\gamma}(\langle S^{\text{st}}\rangle - S_i^t)} \tag{A30}$$

Introducing S_i^{t+1} as given by Eq. (21) and rearranging we obtain

$$e_i^t = \frac{S_i^{\rm st} - S_i^t - \tilde{\gamma}(\langle S^{\rm st}\rangle - S_i^t) + \lambda_2(S_i^t - S_i^{\rm st}) + (\lambda_1 - \lambda_2)(\langle S^t\rangle - \langle S^{\rm st}\rangle)}{\bar{S}_i - S_i^t - \tilde{\gamma}(\langle S^{\rm st}\rangle - S_i^t)} \tag{A31}$$

The above expression corresponds to the effort of worker i at stage t depending in the current preference S_i^t and the long term $S_i^{\rm st}$.

Replacing S_i^t from Eq. (22), we obtain after some tedious but straightforward algebra[15]

$$e_i^t = \frac{\tilde{\gamma}[S_i^{\rm st} - \langle S^{\rm st}\rangle] - \lambda_2^t(1-\tilde{\gamma}+\lambda_2)[S_i^0 - S_i^{\rm st}] - (\lambda_1^{t+1} - \lambda_2^{t+1})(\lambda_1-\lambda_2)(1-\tilde{\gamma})[\langle S^0\rangle - \langle S^{\rm st}\rangle]}{(\bar{S}_i - S_i^{\rm st}) + \tilde{\gamma}[S_i^{\rm st} - \langle S^{\rm st}\rangle] - (1-\tilde{\gamma})(\lambda_2^t(S_i^0 - S_i^{\rm st}) + (\lambda_1^t - \lambda_2^t)[\langle S^0\rangle - \langle S^{\rm st}\rangle)]}.$$

which is Eq. (24) in the main text. It can also be written solely in terms of $\bar{S}_i$ and S_i^0 by replacing $S_i^{\rm st}$ using Eq. (23).

$$e_i^t = \frac{\lambda_2^t(1-\tilde{\gamma}+\lambda_2)[\bar{S}_i - S_i^0] + [\tilde{\gamma}\tilde{d} - \lambda_2^t(1-\tilde{\gamma}+\lambda_2)(1-\tilde{d})][\bar{S}_i - \langle\bar{S}\rangle]}{\lambda_2^t(1-\tilde{\gamma})[\bar{S}_i - S_i^0] + (1-\tilde{d}+\tilde{\gamma}\tilde{d} - \lambda_2^t(1-\tilde{\gamma}))[\bar{S}_i - \langle\bar{S}\rangle] - (\lambda_1^t - \lambda_2^t)(1-\tilde{\gamma})[\langle S^0\rangle - \langle\bar{S}\rangle]}$$
$$\frac{(\lambda_1^{t+1} - \lambda_2^{t+1})(\lambda_1-\lambda_2)(1-\tilde{\gamma})[\langle S^0\rangle - \langle\bar{S}\rangle]}{\lambda_2^t(1-\tilde{\gamma})[\bar{S}_i - S_i^0] + (1-\tilde{d}+\tilde{\gamma}\tilde{d} - \lambda_2^t(1-\tilde{\gamma}))[\bar{S}_i - \langle\bar{S}\rangle] - (\lambda_1^t - \lambda_2^t)(1-\tilde{\gamma})[\langle S^0\rangle - \langle\bar{S}\rangle]}$$

Appendix 2: Properties of matrices $\mathbf{M}_N(a,b)$

Consider the family of $N\times N$ matrices denoted by $\mathbf{M}_N(a,b)$ which are characterized by having a common value, a, at the diagonal and another common value, b, at all non-diagonal entries. Namely,

$$\mathbf{M}_N(a,b) \equiv \begin{pmatrix} a & b & b & \ldots\ldots & b & b \\ b & a & b & \ldots\ldots & b & b \\ \ldots & \ldots & \ldots & \ldots\ldots & \ldots & \ldots \\ b & b & b & \ldots\ldots & a & b \\ b & b & b & \ldots\ldots & b & a \end{pmatrix} \tag{A32}$$

The following summarizes some properties of this set of matrices:
-Eigenvalues:

$$\lambda_1 = a + (N-1)b, \qquad \text{multiplicity } 1, \tag{A33}$$
$$\lambda_2 = a - b, \qquad \text{multiplicity } N-1. \tag{A34}$$

[15] Note that from Eq. (21) we obtain that the average value of the trajectory approaches the steady value exponentially in time, $\langle S^t\rangle = \langle S^{\rm st}\rangle + \lambda_1^t(\langle S^0\rangle - \langle S^{\rm st}\rangle)$.

The eigenvector matrix is:

$$\varphi_N = \begin{pmatrix} 1 & 1 & 1 & \ldots\ldots & 1 & 1 \\ 1 & -1 & 0 & \ldots\ldots & 0 & 0 \\ 1 & 0 & -1 & \ldots\ldots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots\cdots & \cdots & \cdots \\ 1 & 0 & 0 & \ldots\ldots & -1 & 0 \\ 1 & 0 & 0 & \ldots\ldots & 0 & -1 \end{pmatrix} \tag{A35}$$

and all eigenvectors are independent of a and b.
-Determinant: $\text{Det}[\mathbf{M}_N(a,b)] = (a-b)^{N-1}(a+(N-1)b)$.
-Product

$$\begin{aligned} &\mathbf{M}_N(a_1,b_1)\mathbf{M}_N(a_2,b_2) = \mathbf{M}_N(a_2,b_2)\mathbf{M}_N(a_1,b_1) \\ &= \mathbf{M}_N(a_1a_2+(N-1)b_1b_2, a_1b_2+a_2b_1+(N-2)b_1b_2) \end{aligned} \tag{A36}$$

-Powers. If $t \in \mathcal{Z}$, $[\mathbf{M}_N(a,b)]^t = \mathbf{M}_N(a^{(t)}, b^{(t)})$ with

$$a^{(t)} = \frac{(a+(N-1)b)^t + (N-1)(a-b)^t}{N} = \frac{1}{N}\left(\lambda_1^t + (N-1)\lambda_2^t\right) \tag{A37}$$

$$b^{(t)} = \frac{(a+(N-1)b)^t - (a-b)^t}{N} = \frac{1}{N}\left(\lambda_1^t - \lambda_2^t\right) \tag{A38}$$

-Particular case, inverse matrix

$$[\mathbf{M}_N(a,b)]^{-1} = \mathbf{M}_N(a^{(-1)}, b^{(-1)}) \tag{A39}$$

with

$$a^{(-1)} = \frac{a+(N-2)b}{(a-b)(a+(N-1)b)} \tag{A40}$$

$$b^{(-1)} = \frac{-b}{(a-b)(a+(N-1)b)} \tag{A41}$$

-Multiplication by a vector: $[\mathbf{M}_N(a,b)x\,]_i = (a-b)x_i + Nb\,\langle x\rangle$.

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} \tag{A42}$$

$$X^T = (x_1, \cdots, x_N) \tag{A43}$$

$$\langle XY\rangle = \frac{1}{N}\sum_{i=1}^{N} x_i y_i = \frac{1}{N}X^T Y \tag{A44}$$

$$\tag{A45}$$

This gives:

$$\langle MX \rangle = \lambda_1 \langle X \rangle \tag{A46}$$

$$\langle X(MY) \rangle = \frac{1}{N} X^T MY = (\lambda_1 - \lambda_2) \langle X \rangle \langle Y \rangle + \lambda_2 \langle XY \rangle \tag{A47}$$

These formulas are to be applied with $M = \Pi_1^t \Pi_2^{t'}$ using the appropriate values of λ_1 and λ_2, and $X, Y = C_1, C_2$.